

where  $T$  is the absolute temperature. In the iteration procedure followed, new values for  $C_0$  and  $s$  were generated for each successive repetition; these values were used in turn to evaluate  $\gamma(\rho)$  in the following iteration. The procedure was simply repeated until convergence was attained.

The foregoing procedure was initially carried out for values of the Anderson-Grüneisen parameter ( $\delta$ ) equal to 4, 6, 8 and 10. The resulting volume dependence of the Grüneisen parameter for each case is indicated in Fig. 2. Based on these calculated curves, a value of  $\delta = 7 \pm 1$  was determined by minimizing the standard deviation of the porous and fused-quartz experimental data. This value of  $\delta$  was then used for the final calculation of the elastic properties of stishovite using again the described iteration procedure. The results are:  $K_0^S = 3.35 \pm 0.19$  Mbar;  $(\partial K^S/\partial P)_T = 5.5 \pm 0.6$ ; and  $(\partial K^S/\partial T)_P = -0.35 \pm 0.08$  Kbar/ $^\circ K$ .

The values of  $K_0^S$  and  $(\partial K^S/\partial P)_T$  are based on the Hugoniot data of  $\alpha$ -quartz from separate studies by Wackerle (1962), Al'tshuler *et al.* (1965), and Trunin *et al.* (1971). In order to weight the data from each of these sources appropriately for the final calculations, each set initially was reduced independently. Individual standard deviations in the  $U_S - u_p$  plane were calculated thereby for each source; these formed the basis for weighting the various data sets in the final analysis. The metastable Hugoniot data for stishovite in the  $U_S - u_p$  plane, and the linear fit providing the final elastic properties listed above, are indicated in Fig. 5. The precision in the data of Al'tshuler *et al.* (1965) and Trunin *et al.* (1971) is clearly superior to that of Wackerle (1962); the data were weighted in accordance with this observation.

The possibility exists that systematic error in the elastic properties of stishovite calculated from shock-wave data may result from an inherent 'strength effect'. This effect would occur if the material was shocked to a state along the deformational

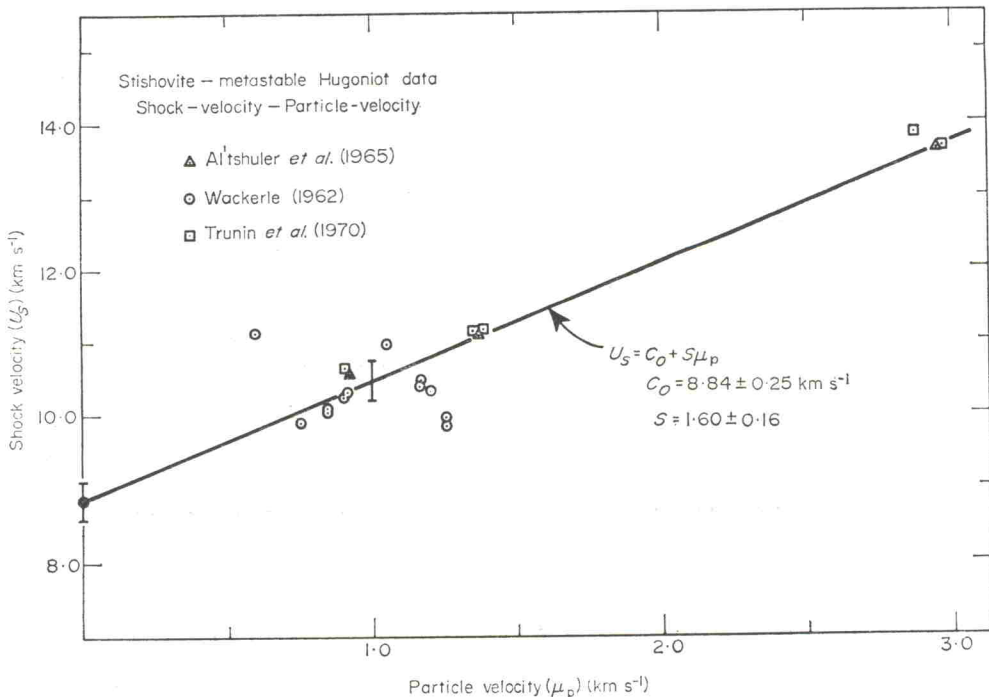


FIG. 5. Shock-velocity-particle-velocity metastable Hugoniot data for  $\alpha$ -quartz and quartzite in the stishovite regime. The final linear least-square fit and errors are indicated.

Hugoniot, and was still able to support a finite shear stress. However, it has been suggested by Ahrens *et al.* (1969), that because the fundamental crystal structure undergoes complete rearrangement during a shock-induced phase transformation, it appears unlikely that an appreciable shear strength would characterize the high-pressure phase. In the case of  $\alpha$ -quartz, shock-Hugoniot measurements to 230 Kbars by Fowles (1967) have demonstrated that the cohesion of the material apparently is destroyed upon yielding at the Hugoniot elastic limit (HEL); thus, there is no indication of a residual shear stress. Therefore, the assumption that shear strength offsets are negligible in the Hugoniot data of stishovite appears quite reasonable.

### Equation of state considerations

The equation of state is of critical importance with respect to compositional aspects of the mantle. Several forms have been utilized to predict the compression of various materials to pressures characteristic of the lower mantle from low-pressure ultrasonic data (e.g. O. L. Anderson 1966; and Ringwood 1970); in addition, Anderson & Jordan (1970) have fit lower mantle pressure-density models directly to a particular equation of state in order to derive elastic properties characteristic of that region. Clearly, it is of significant importance to determine and understand the function of the form of the equation of state in the construction of meaningful compositional models of the Earth's interior. Because, there is abundant evidence that suggests the occurrence of silicon in the lower mantle in 6-fold co-ordination with oxygen, the equation of state of stishovite is of special interest.

A number of theoretical and empirical expressions have been proposed for the description of the equation of state of solids at high pressure. At present, theoretical equations of state contain a high degree of uncertainty because of the simplifying assumptions necessary for a tractable solution. Therefore, empirical equations have been used most extensively for geophysical applications. Most of these equations are based on a truncated Taylor expansion and can therefore be expressed in terms of an arbitrary number of parameters, corresponding to the order of the approximation. A most important implication of the use of such empirical expressions has been pointed out by Barsch & Chang (1970); since they are 'phenomenological' equations, for a given number of parameters, which form is most appropriate for an accurate representation of experimental data over a specified pressure range can only be determined *a posteriori*, i.e. by comparison with the experimental data. This observation also has been noted by MacDonald (1969) who theoretically and statistically compared several polynomial and non-linear equations of state with experimental data sets. In this section, the results of the present analysis, incorporating the first-order Murnaghan equation of state, are compared with static-compression and ultrasonic data characterizing the zero to 250 Kbar range. In addition, the combined shock-wave, static-compression, and ultrasonic results are discussed in terms of the form of the equation of state; specifically, the shock-wave analysis of Ahrens *et al.* (1970) using the first-order Birch equation, and that of Davies (1972) involving a 'fourth-order Eulerian' equation, are compared with the present results.

The isentrope defined by the present results according to the first-order Murnaghan equation of state

$$\rho(P) = \rho_0 \left[ 1 + \frac{\beta^S P}{K^S} \right]^{1/\beta^S} \quad (8)$$

where  $\beta^S = (\partial K^S / \partial P)_S$ , is indicated in Fig. 1. Clearly, the elastic properties of stishovite, independently derived from the shock-wave Hugoniot data using the Murnaghan equation, are quite consistent with the preferred static compression data of Liu *et al.* (1971), to approximately 250 Kbar, as well as with the ultrasonic measurements of